Schwartz
8.3

massive spin-1 Lagrangian, with no current:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}m^2 A_\mu^2$$

with current:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}m^2 A_\mu^2 - J_\mu A_\mu.$$

If the eqm for $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - J_\mu A_\mu$ is $\partial_\mu F_{\mu\nu} = J_\nu$,

then the eqm for this Lagr. would be

$$m^2 A_\nu - J_\nu + \partial_\mu F_{\mu\nu} = 0,$$

$$\boxed{m^2 A_\nu + \partial_\mu F_{\mu\nu} = J_\nu}$$

In momentum space, it's

$$m^2 A_\nu \cancel{+} - p_\mu (p_\mu A_\nu - p_\nu A_\mu) = J_\nu$$

$$m^2 g_{\mu\nu} A_\mu \cancel{+} - p^2 g_{\mu\nu} A_\mu + p_\mu p_\nu A_\mu = J_\nu$$

$$\left[(m^2 \pm p^2) g_{\mu\nu} + p_\mu p_\nu\right] A_\mu = J_\nu$$

$$\left[(m^2 - p^2) g_{\mu\nu} + p_\mu p_\nu\right] A_\mu = J_\nu$$

Invert $[(m^2-p^2)g_{\mu\nu}+p_\mu p_\nu]\, p_\mu p_\nu$

$$= [(m^2-p^2)p^2+p^4]$$

$$= m^2p^2$$

$\Rightarrow$

$$[(m^2-p^2)g_{\mu\nu}+p_\mu p_\nu]\,\frac{p_\mu p_\nu}{m^2p^2} = 1$$

,

$$[(m^2-p^2)g_{\mu\nu}+p_\mu p_\nu]^{-1} = \frac{p_\mu p_\nu}{m^2p^2},$$

$A_\mu = \dfrac{p_\mu p_\nu}{m^2p^2} J_\nu$, in form of $A_\mu = \Pi_{\mu\nu} J_\nu$

$$\Rightarrow \boxed{\Pi_{\mu\nu} = \frac{p_\mu p_\nu}{m^2p^2}}$$

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